Olimpiada de matematică

Bistrita- Etapa județeană- 12.03.2011

Clasa a VI-a

Problema 1

$8+8\cdot 9+8\cdot 9^2+\ldots+8\cdot 9^{2010}=8(\underbrace{1+9+9^2+\ldots+9^{2010}}_{S})$ 1p

$$-\begin{cases} S=1+9+9^2+\ldots+9^{2010} \;/\cdot 9 \\ 9S=9+9^2+\ldots+9^{2010}+9^{2011} \end{cases}$$

$8S=9^{2011}-1$ 1p

$8S=x^{2011}-1$

$9^{2011}-1=x^{2011}-1 \Rightarrow x=9$ 1p

$A=9+99+999+\ldots+\overline{\underbrace{999\ldots9}_{2011\,cifre}}$

$A=(10-1)+(100-1)+(1000-1)+\ldots+(\overline{1\underbrace{00000\ldots0}_{2011\,cifre\,de\,0}}-1)$ 1p

$A=10+100+\ldots+\overline{1\underbrace{00000\ldots0}_{2011\,cifre\,de\,0}}-(\underbrace{1+1+\ldots+1}_{2011})$

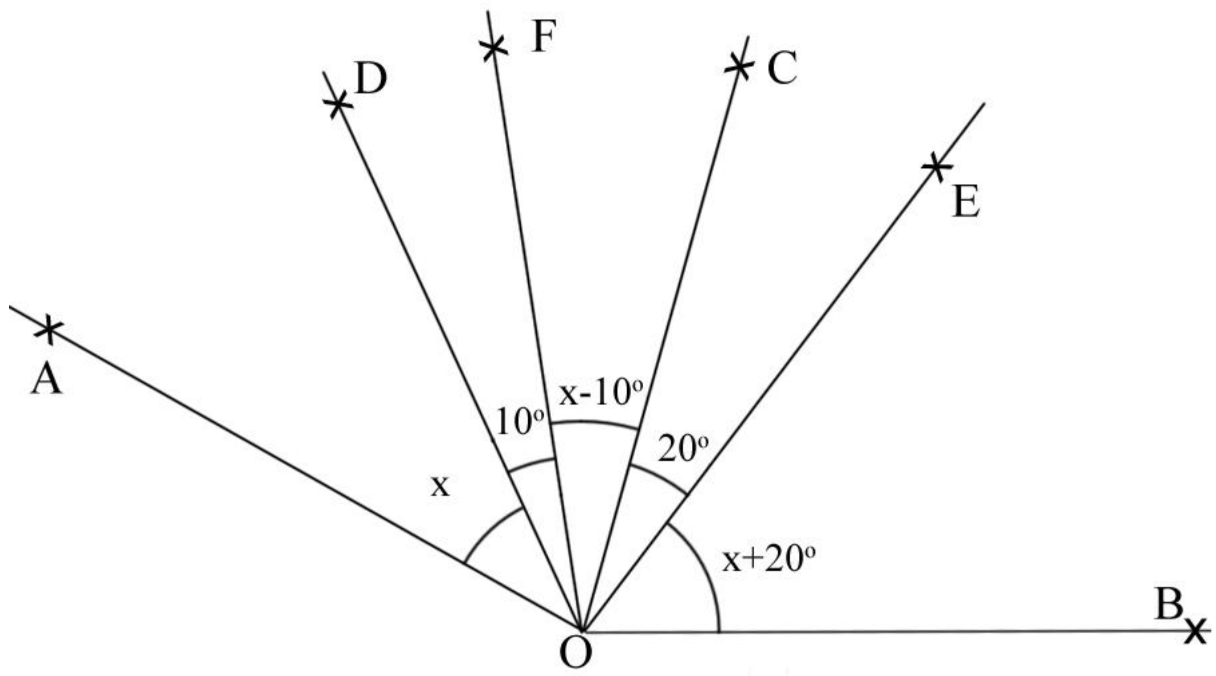
$A=11\ldots10-2011$ 1p

$$\begin{array}{r} 111\ldots111110\;- \\ 2011 \\ \hline \underbrace{111\ldots1}_{2007}09099 \end{array}$$

$A=11\ldots109099$ 1p

$Suma\,cifrelor=2007+27=2034$ 1p

Problema 2



$m(\angle AOB) \overset{not}{=} x$ 1p

$m(\angle AOF) = x + 10^o$
$m(\angle AOD) = m(\angle DOC)$
$m(\angle FOC) = x - 10^o$ 2p

$m(\angle AOF) = m(\angle FOE) <=> x + 10^o = x - 10^o + m(\angle EOC) =>$

$=> m(\angle EOC) = 20^o$ 1p

$m(\angle BOE) = m(\angle EOD) <=> m(\angle BOE) = 10^o + x - 10^o + 20^o$ 1p

$m(\angle AOD) + m(\angle DOF) + m(\angle FOC) = m(\angle COE) + m(\angle BOE)$
$x + 10^o + x - 10^o = 20^o + x + 20^o$
$2x = 40^o + x$
$x = 40^o$ 1p

Deci $m(\angle AOB) = (40^o + 20^o + 20^o) \cdot 2 = 160^o$ 1p

Problema 3

$\frac{1}{2} < \frac{2}{3}$
$\frac{3}{4} < \frac{4}{5}$
....
$\frac{99}{100} < \frac{100}{101}$ 2p

$P < P_1 \ / \cdot P \ => \ P^2 < P \cdot P_1$ 1p

$P \cdot P_1 = \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \ldots \cdot \frac{99}{100} \cdot \frac{2}{3} \cdot \frac{4}{5} \cdot \ldots \cdot \frac{98}{99} \cdot \frac{100}{101} = \frac{1}{101}$ 1p

$P \cdot P_1 = \frac{1}{101} < \frac{1}{100}$

$P^2 < P \cdot P_1$
$P \cdot P_1 < \frac{1}{100}$ $=> P^2 < \frac{1}{100}$ 2p

Deci $P < \frac{1}{10}$ 1p